Properties of Laplace Transform.

(1) $L\{a f_1(t) \pm b f_2(t)\} = a\,L\{f_1(t)\} \pm b\,L\{f_2(t)\}$

$$\Rightarrow \int_0^\infty e^{-st}[a f_1(t) \pm b f_2(t)]\,dt = a\int_0^\infty e^{-st} f_1(t)\,dt \pm b\int_0^\infty e^{-st} f_2(t)\,dt$$

↳ Linear Property.

(2) Change of Scale Property.

If $L\{f(t)\} = F(s)$ then $\boxed{L\{f(at)\} = \frac{1}{a}F\left(\frac{s}{a}\right)}$

$$\Rightarrow L\{f(at)\} = \int_0^\infty e^{-st}\cdot f(at)\,dt$$

$at = u$.
$a\,dt = du$
$t=0 \Rightarrow u=0$
$t=\infty \Rightarrow u=\infty$

$$= \int_0^\infty e^{-\frac{su}{a}}\cdot f(u)\,\frac{du}{a}$$

$$= \frac{1}{a}\int_0^\infty e^{-Su}\cdot F(u)\,du \qquad S = \frac{s}{a}$$

$$= \frac{1}{a}F(S) = \frac{1}{a}F\left(\frac{s}{a}\right).$$

so, $\boxed{L\{f(at)\} = \frac{1}{a}F\left(\frac{s}{a}\right)}$

Exp:. $L\{(6t)^4\} = ?$

as, $L\{t^4\} = \frac{4!}{s^5}$ then, $L\{(6t)^4\} = \frac{1}{6}\cdot\frac{4!}{\left(\frac{s}{6}\right)^5} = \frac{6^5\cdot 4!}{6\cdot s^5} = \frac{6^4\cdot 4!}{s^5}$

(3) If $f(t)$ is defined for $t>0$ and $f(t)$ is sectionally continuous and of exponetical order then $F(s) \longrightarrow 0$ as $s \to \infty$

(4) First shifting property:-

If $L\{f(t)\} = F(s)$

then $L\{e^{at} f(t)\} = F(s-a)$

$$\Rightarrow \quad L\{e^{at} f(t)\} = \int_0^\infty e^{-st} \cdot e^{at} \cdot f(t)\, dt$$

$$= \int_0^\infty e^{-(s-a)t} \cdot f(t)\, dt$$

$$= \underline{F(s-a)}.$$

Exp:- $L\{e^{2t} \cdot \sin 4t\}$.

as, $L\{\sin 4t\} = \frac{4}{s^2+(4)^2} = \frac{4}{s^2+16}$

so, by shifting property -

$$L\{e^{2t} \sin 4t\} = \frac{4}{(s-2)^2+16}.$$

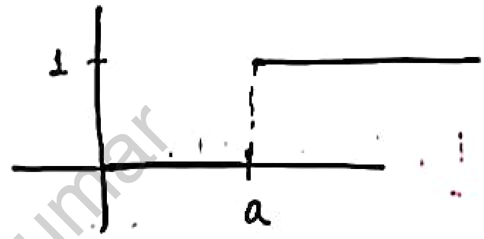
(5)

Unit Step function (Heaviside's).

$$U(t) = \begin{cases} 0 & t<0 \\ 1 & t \geq 0 \end{cases}$$

a.

$$U(t-a) = \begin{cases} 0 & 0<t<a \\ 1 & t \geq a \end{cases}$$



Now, $\boxed{L\{U(t-a)\} = \frac{e^{-sa}}{s}}$

sol

$$L\{U(t-a)\} = \int_0^\infty e^{-st} \cdot U(t-a) \cdot dt$$

$$= \int_0^a e^{-st} \times 0 \times dt + \int_a^\infty e^{-st} \cdot 1 \, dt$$

$$= \left[\frac{e^{-st}}{-s}\right]_a^\infty$$

$$= \left[0 + \frac{e^{-as}}{s}\right] = \frac{e^{-as}}{s}$$

* (6).

If $L\{f(t)\} = F(s)$ (Second shifting Property)

and $g(t) = \begin{cases} f(t-a) & t>a \\ 0 & 0<t<a \end{cases}$

then, $\boxed{L\{g(t)\} = e^{-as} \cdot F(s)}$